

Time : 3 Hours]

[Total Marks : 60

Instructions to the Students:

1. Each question carries 12 marks.
2. Question No. 1 will be compulsory and include objective-type questions.
3. Candidates are required to attempt any four questions from Question No. 2 to Question No.6
4. Use of non-programmable scientific calculators is allowed.
5. Assume suitable data wherever necessary and mention it clearly.

Q1. Objective type questions. (Compulsory Question)

12

1

Laplace Transform of $\sinh 3t$ is

a. $\frac{s}{s^2-9}$

b. $\frac{3}{s^2+9}$

c. $\frac{s}{s^2-3}$

d. $\frac{3}{s^2-9}$

2

Laplace Transform of t^3

a. $\frac{2!}{s^3}$

b. $\frac{2!}{s^4}$

c. $\frac{3!}{s^3}$

d. $\frac{3!}{s^4}$

3

 $L\{u(t-4)\} =$

a. e^{4s}

b. e^{-4s}

c. $\frac{e^{-4s}}{s}$

d. $\frac{e^{4s}}{s}$

4

 $L^{-1}\left\{\frac{1}{(s+a)^2}\right\} =$

a. te^{-at}

b. te^{at}

c. $-te^{-at}$

d. $-te^{at}$

5

 $L^{-1}\left\{\frac{s^2-3s+4}{s^3}\right\} =$

a. $1-3t-2t^2$

b. $1+3t+2t^2$

c. $1-3t+2t^2$

d. None

6

 $L^{-1}\left\{\log\left(\frac{s+a}{s+b}\right)\right\} =$

a. $\frac{e^{bt}-e^{at}}{t}$

b. $\frac{e^{-bt}+e^{-at}}{t}$

c. $\frac{e^{-bt}-e^{-at}}{t}$

d. None

7

Find the Fourier sine transform e^{-x} is

- a. $\frac{s}{s^2+1}$ b. $\frac{s}{s^2-1}$ c. $\frac{1}{s^2+1}$ (d) None

8

If $F_c\{f(x)\} = F(s)$ then $F_c\{f(ax)\} = kF\left(\frac{s}{a}\right)$, what is the value k?

- a. $\frac{2}{a}$ b. $\frac{s}{a}$ c. $\frac{1}{a}$ d. a

9

One dimensional heat flow equation is

- a. $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ b. $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ c. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ d. None

10

Which of following is Partial differential equation form by eliminating arbitrary constants a & b from $z = ax + by$

- a. $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0$ b. $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z$ c. $y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = 0$ d. None

11

For what value of m the function $f(z) = x^2 - y^2 + mxyi$ is analytic

- a. $m = 1$ b. $m = -1$ c. $m = 2$ d. $m = -2$

12

The value of $\int_c \frac{z}{z^2+2z-8} dz$ where c is circle $|z| = \frac{3}{2}$

- a. $2\pi i$ b. $4\pi i$ c. 0 d. None

Q2. Solve the following.

A)

Evaluate using Laplace Transform $\int_0^\infty \frac{\cos at - \cos bt}{t} dt$

6

B)

Find Laplace Transform of $\int_0^t te^{-t} \sin 4t dt$

6

Q3. Solve the following.

A)

Find Inverse Laplace Transforms by convolution theorem $\frac{s^2}{(s^2+4)^2}$

6

B)

Solve using Laplace Transform

6

$$\frac{d^2 y}{dt^2} - 4 \frac{dy}{dt} + 3y = e^{-t}; y(0) = 1 \text{ and } y'(0) = 0$$

Q4. Solve Any Two of the following.

A) Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ as Fourier integral 6

Hence prove that $\int_0^\infty \frac{\sin t}{t} dt = \frac{\pi}{2}$

B) Find Fourier Transform of $f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ 6

Hence evaluate $\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx$

C) Using Parseval's identity, prove that $\int_0^\infty \frac{t^2}{(4+t^2)(9+t^2)} dt = \frac{\pi}{10}$ 6

Q5. Solve Any Two of the following.

A) Solve: $(y + zx)p - (x + yz)q = x^2 - y^2$ 6

B) Form of Partial Differential Equation from the following equation by elimination of arbitrary function from $f(x + y + z, x^2 + y^2 + z^2) = 0$ 6

C) Use Method of separation of variable to solve the equation 6

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u, \text{ given that } u(x, 0) = 6e^{-3x}$$

Q6. Solve Any Two of the following.

A) Find the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y + \cos 2x}$ 6

B) Evaluate $\oint \frac{\sin^6 z}{(z - \frac{\pi}{2})^3} dz$ where c is $|z| = 2$ 6

C) Using Cauchy's residue theorem, Evaluate $\oint_c \frac{1-2z}{z(z-1)(z-2)} dz$ where c is $|z| = \frac{3}{2}$ 6

*** End ***